



### FREELAND CE PRIMARY SCHOOL CALCULATION POLICY

### **Mathematics Mastery**

At the centre of the mastery approach to the teaching of mathematics is the belief that all children have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly, with calculation strategies, children must not simply rote learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations. This policy outlines the different calculation strategies that should be taught and used in Year 1 to Year 6 in line with the requirements of the 2014 Primary National Curriculum.

### Mathematical Language

The 2014 National Curriculum is explicit in articulating the importance of children using the correct mathematical language as a central part of their learning (reasoning). Indeed, in certain year groups, the non-statutory guidance highlights the requirement for children to extend their language around certain concepts. It is therefore essential that teaching using the strategies outlined in this policy is accompanied by the use of

appropriate and precise mathematical vocabulary. New vocabulary should be introduced in a suitable context (for example, with relevant real objects, apparatus, pictures or diagrams) and explained carefully. High expectations of the mathematical language used are essential, with teachers only accepting what is correct.

The quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof.

2014 Maths Programme of Study

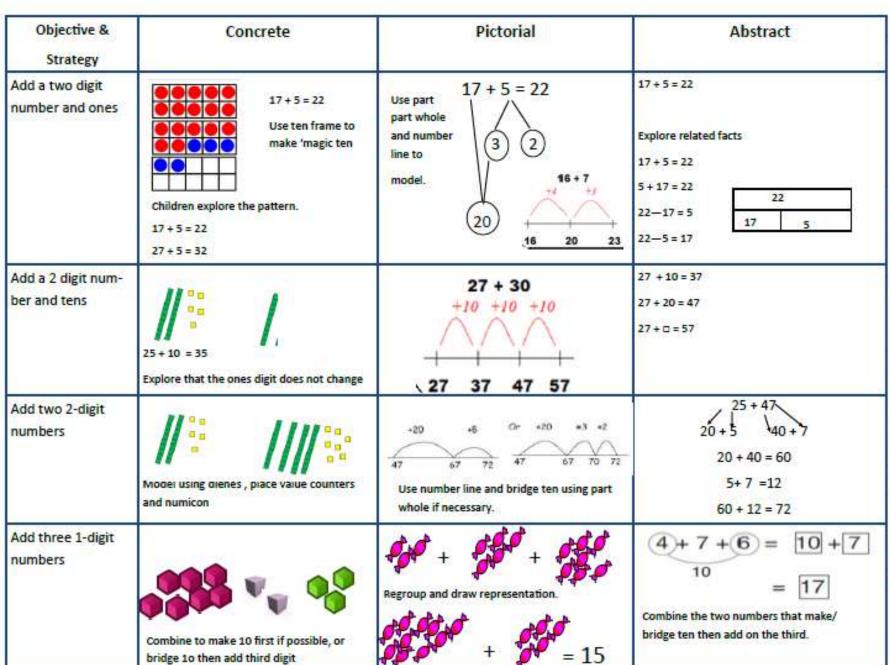
### How to use the policy

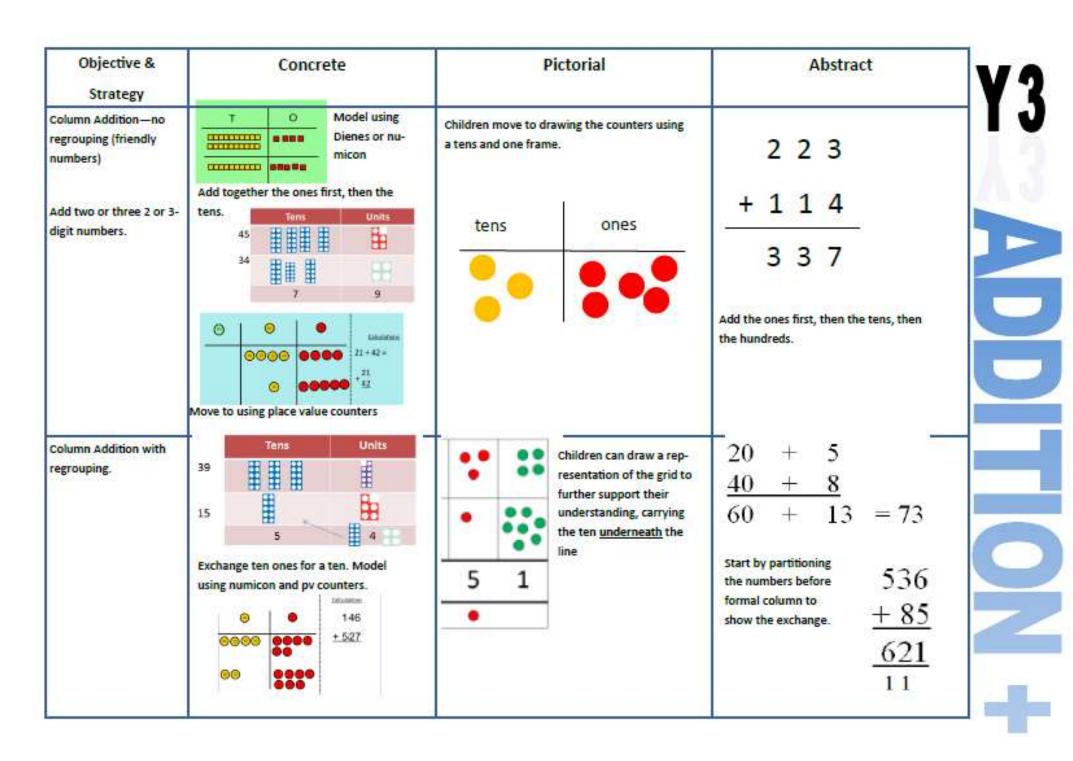
This mathematics policy is a guide for all staff at Freeland CE Primary school and has been adapted from work by the NCETM. It is purposely set out as a progression of mathematical skills and year group phases but a flexible approach to teaching and learning is needed according to the cohort and individual needs. It is expected that teachers will use their professional judgement as to when consolidation of existing skills is required or if to move onto the next concept. However, the focus must always remain on breadth and depth rather than accelerating through concepts. Children should not be extended with new learning before they are ready, they should deepen their conceptual understanding by tackling challenging and varied problems. All teachers use the scheme of work from the White Rose Maths Hub and are required to base their planning around their year groups modules and not to move onto a higher year groups scheme of work. Teachers can use any teaching resources that they wish to use and the policy does not recommend one set of resources over another, rather that, a variety of resources are

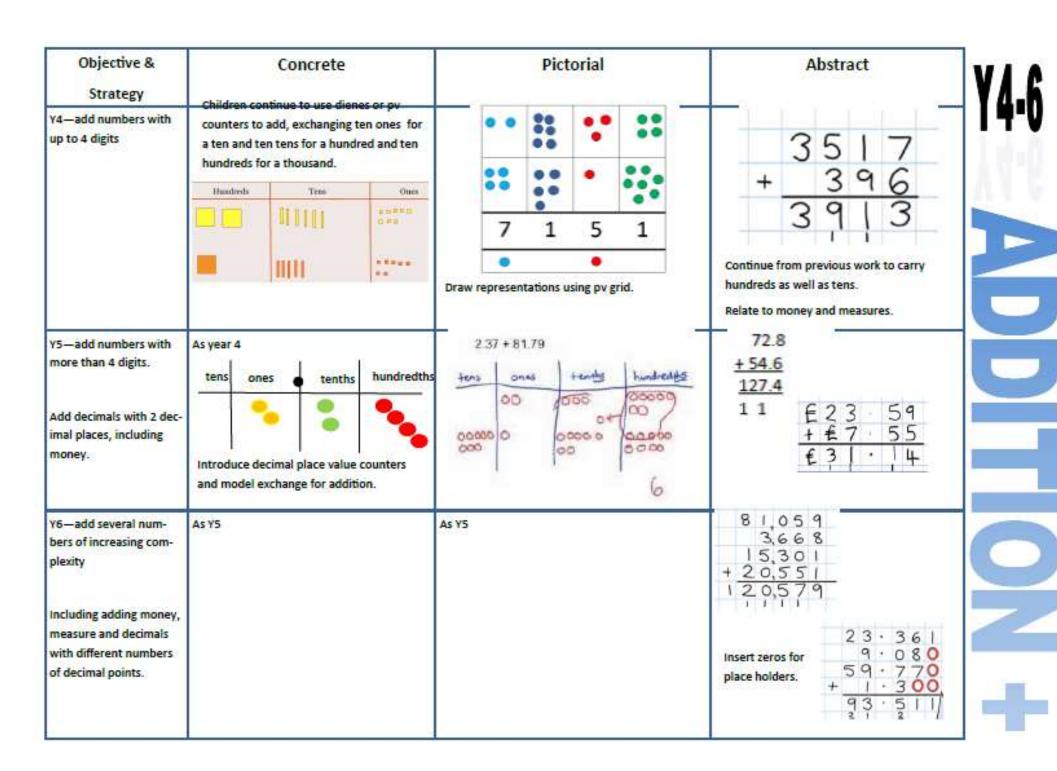
used. For each of the four rules of number, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations. The principle of the concrete-pictorial-abstract (CPA) approach [Make it, Draw it, Write it] is for children to have a true understanding of a mathematical concept, they need to master all three phases within a year group's scheme of work.

Objective & Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	Use part part whole model. Use cubes to add two numbers together as a group or in a bar.	Use pictures to add two numbers together as a group or in a bar.	4 + 3 = 7  Use the part-part whole diagram as shown above to move into the abstract.
Starting at the big- ger number and counting on	Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  10 11 12 13 14 15 16 17 18 19 20  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17  Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10.  This is an essential skill for column addition later.	Start with the bigger number and use the smaller number to make 10.  Use ten frames.	3 + 9 =  Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10.  9 + 5 = 14	7 + 4= 11  If I am at seven, how many more do I need to make 10. How many more do I add on now?
Represent & use number bonds and related subtraction facts within 20	2 more than 5.	Constitution (Management	Emphasis should be on the language '1 more than 5 is equal to 6.' '2 more than 5 is 7.' '8 is 3 more than 5.'

Objective & Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	50= 30 = 20  Model using dienes and bead strings	O tower + 5 tone tone 30 + 50 =  Use representations for base ten.	20 + 30 = 50 70 = 50 + 20 40 + = = 60
Use known number facts Part part whole	Children explore ways of making numbers within 20	20	+ 1 = 16
Using known facts		∴ + ∴ = ∴	3 + 4 = 7 leads to 30 + 40 = 70 leads to 300 + 400 = 700
Bar model	3+4=7	7+3=10	23 25 ? 23 + 25 = 48







Objective & Strategy	Concrete	Pictorial	Abstract
Taking away ones.	Use physical objects, counters, cubes etc to show how objects can be taken away.  6—4 = 2		7—4 = 3
	4-2=2	15 – 3 = 12  Cross out drawn objects to show what has been taken away.	16—9 = 7
Counting back	Move objects away from the group, counting backwards.  Move the beads along the bead string as you count backwards.	Count back in ones using a number line.	Put 13 in your head, count back 4. What number are you at?
ind the Difference	Compare objects and amounts  7 'Seven is 3 more than four'  4 'I om 2 years older than my sister'	Count on using a number line to find the difference.	Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister.?
	Sister Si	0 1 2 3 4 5 6 7 8 9 10 11 12	

Objective & Strategy	Concrete	Pictorial	Abstract
Represent and use number bonds and related subtraction facts within 20 Part Part Whole model	Link to addition. Use PPW model to model the inverse.  If 10 is the whole and 6 is one of the arts, what s the other part?  10—6 = 4	Use pictorial representations to show the part.	Move to using numbers within the part whole model.  5 12 7
Make 10	Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.	13—7  13—7 = 6  Jump back 3 first, then another 4. Use ten as the stopping point.	16—8  How many do we take off first to get to 10? How many left to take off?
Bar model	5-2=3	3333333 33	8 2 10 = 8 + 2 10 = 2 + 8 10-2 = 8

**Y1** BIR 

Concrete	Pictorial	Abstract
Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'	20 – 4 =	20—4 = 16
Use Dienes to show how to partition the number when subtracting without regrouping.	Children draw representations of Dienes and cross off.	43—21 = 22
34—28 Use a bead bar or bead strings to model counting to next ten and the rest.	75 80 90 93 'counting on' to find difference'  Use a number line to count on to next ten and then the rest.	93—76 = 17
	Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'  34—13 = 21  Use Dienes to show how to partition the number when subtracting without regrouping.	Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'  34—13 = 21  Use Dienes to show how to partition the number when subtracting without regrouping.  34—21 = 22  Use a bead bar or bead strings to model  Use a bead bar or bead strings to model

Objective & Strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping (friendly numbers)	47—32 Use base 10 or Numicon to model	Darw representations to support under- standing	$47-24=23$ $-\frac{20+4}{20+3}$ Intermediate step may be needed to lead to clear subtraction understanding.
Column subtraction with regrouping	Tens Units	45 -29 Tens 10nes 16	836-254*582 Begin by partitioning into pv columns  836-254*582  Begin by partitioning into pv columns
	Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into tten ones. Use the phrase 'take and make' for exchange.	Children may draw base ten or PV counters and cross off.	7 28 - 582 = 146 Then move to formal method.

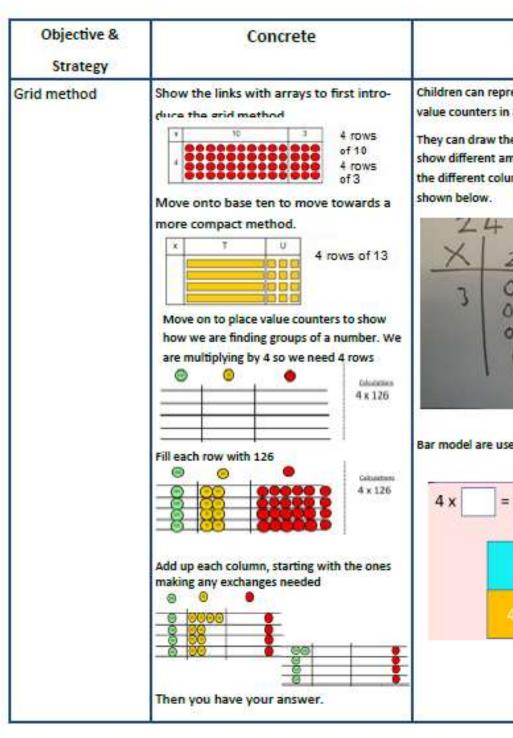
Objective & Strategy		Conc	crete	Pictorial	Abstract
Subtracting tens and ones  Year 4 subtract with up to 4 digits.  Introduce decimal subtrac- tion through context of money	⊕ ⊚  Model proc	@ @ @ @ @ @ @ @ @ @ @ @ @ @ @ @ @ @ @	179	Children to draw pv counters and show their exchange—see Y3	2 X 5 4 - 1 5 6 2 1 1 9 2 Use the phrase 'take and make' for exchange
Year 5- Subtract with at least 4 dig- its, including money and measures. Subtract with decimal values, including mixtures of integers and decimal and aligning the decimal	As Year 4			Children to draw pv counters and show their exchange—see Y3	"8" X '0 8 '6 - 2 1 2 8 2 8,9 2 8 Use zeros for place- holders 3 7 2 · 5 6 7 9 6 · 5
Year 6—Subtract with increasingly large and more complex numbers and decimal values.					1816,699 - 89,949 - 60,750 - 36.080 - 36.080

# SUBTRACTION -

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipultives including cubes and Numicon to demonstrate doubling	Double 4 is 8	Partition a number and then double each part before recombining it back together.  16  10 6  1x2 1x2 20 + 12 = 32
Counting in multi- ples	Count the groups as children are skip counting, children may use their fingers as they are skip counting.	Children make representations to show counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers.  2, 4, 6, 8, 10  5, 10, 15, 20, 25, 30
Making equal groups and counting the total	□ x = 8 Use manipulatives to create equal groups.	Draw to show 2 x 3 = 6  Draw and make representations	2 x 4 = 8

Objective & Strategy	Concrete	Pictorial	Abstract
Repeated addition	Use different objects to add equal groups	Use pictorial including number lines to solve prob  There are 3 sweets in one bag.  How many sweets are in 5 bags altogether?  3+3+3+3+3 = 15	Write addition sentences to describe objects and pictures.  2+2+2+2 + 2 = 10
Understanding ar- rays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.	Draw representations of arrays to show under- standing	3 x 2 = 6 2 x 5 = 10

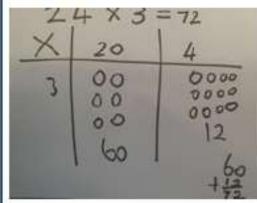
Objective & Strategy	Concrete	Pictorial	Abstract ¥2
Multiplication is commutative	Create arrays using counters and cubes and Numicon.  Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.	Use representations of arrays to show different calculations and explore commutativity.	12 = 3 × 4  12 = 4 × 3  Use an array to write multiplication sentences and reinforce repeated addition  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
Using the Inverse This should be taught alongside division, so pupils learn how they work alongside each other.		8   x   =	2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8 ÷ 2 Show all 8 related fact family sentences.



### Pictorial

Children can represent their work with place value counters in a way that they understand.

They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.



Bar model are used to explore missing numbers

### Abstract

Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

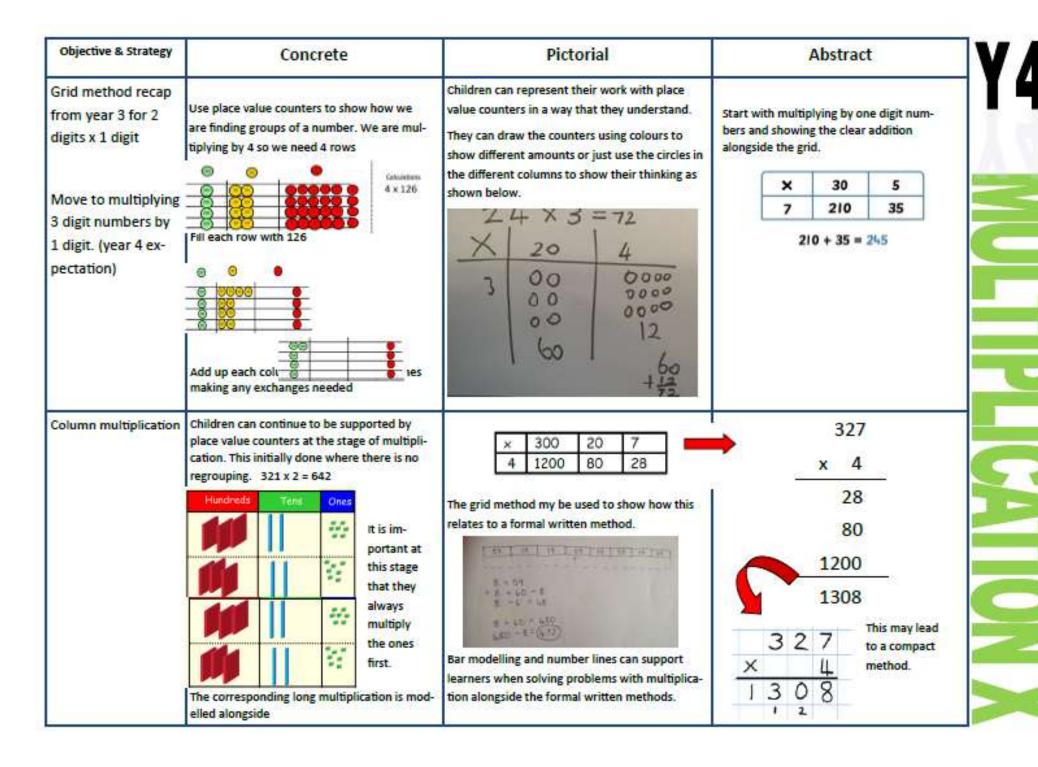
×	30	5
7	210	35

$$210 + 35 = 245$$

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

	10	8
10	100	80
3	30	24





Objective & Strategy	Concrete	Pictorial	Abstract	<b>Y5.6</b>
Column Multiplication for 3 and 4 digits x 1 digit.	Hundreds  It is important at this stage that they always multiply the ones first.  Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642	x 300 20 7 4 1200 80 28	327 x 4 28 80 1200 1308 3 2 7 X 4 1 3 0 8 This will lead to a compact method.	
Column multiplication	Manipulatives may still be used with the corresponding long multiplication modelled alongside.	10 8 80 3 30 24 Continue to use bar modelling to support problem solving	1 8 18 x 3 on the first row  (8 x 3 = 24, carrying the 2 for 20, then 1 x 3)  1 8 0 then 1 x 3)  1 8 x 10 on the 2nd row. Show multiplying by 10 by putting 2ero in units first  1 2 3 4 0 (1234 x 10)  1 9 7 4 4	

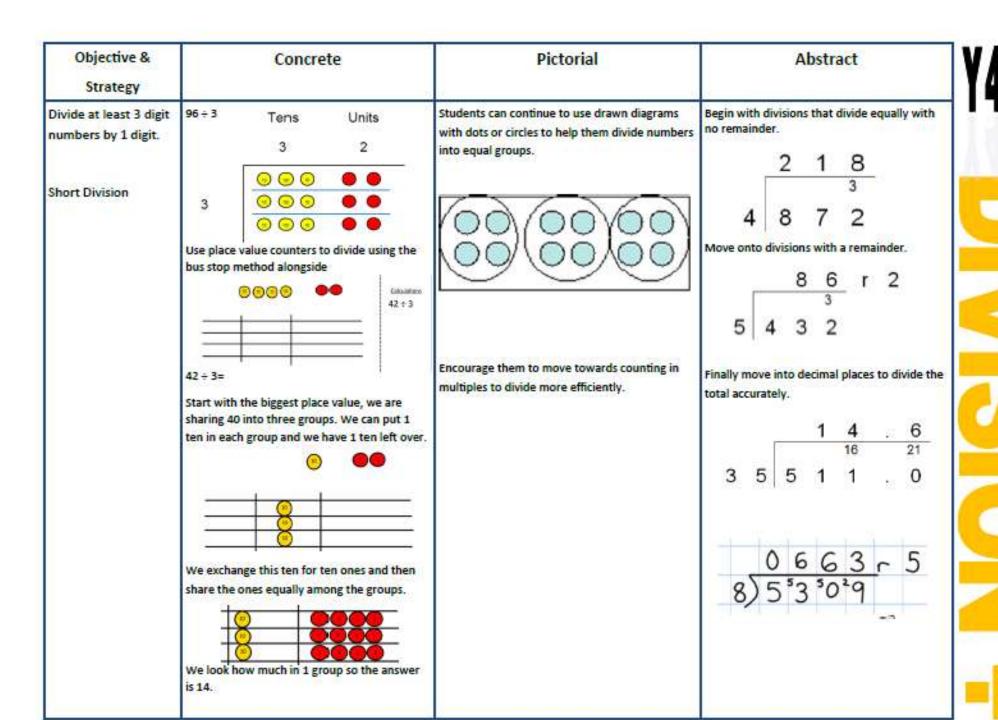
Objective &	Concrete	Pictorial	Abstract
Strategy			
Multiplying decimals			Remind children that the single digit belongs
up to 2 decimal plac- es by a single digit.			in the units column. Line up the decimal points in the question and the answer.
es by a single digit.			
			3 · 1 9
			× 8
			2 5 5 2
			25.52

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	6.6	Children use pictures or shapes to share quantities.	12 shared between 3 is
Use Gordon ITPs for modelling		S Shared Detween 2 is 4	
		Sharing:  4  4  12 shared between 3 is 4	
	10		
	I have 10 cubes, can you share them equally in 2 groups?		

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4  Children use bar modelling to show and support understanding.	12 ÷ 3 = 4
Division as grouping	Divide quantities into equal groups.  Use cubes, counters, objects or place value counters to aid understanding.	Use number lines for grouping  12 + 3 = 4  Think of the part as a whole, split it must the number of groups you are dividing by and work out how many would be within each group.	28 ÷ 7 = 4  Divide 28 into 7 groups. How many are in each group?

Objective & Strategy	Concrete	Pictorial Pictorial	Abstract
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of $6 = 4$ 96 ÷ 3 = 32	Continue to use bar modelling to aid solving division problems.  20  20 + 5 = ? 5 x ? = 20	How many groups of 6 in 24? 24 ÷ 6 = 4
Division with arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created.  Eg 15 ÷ 3 = 5 5 x 3 = 15  15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences	Find the inverse of multiplication and division sentences by creating eight linking number sentences.  7 x 4 = 28  4 x 7 = 28  28 ÷ 7 = 4  28 ÷ 4 = 7  28 = 7 x 4  28 = 4 x 7  4 = 28 ÷ 7  7 = 28 ÷ 4

Objective & Strategy	Concrete	Pictorial	Abstract
Division with remainders.	Divide objects between groups and see how much is left over  Example witho 40 + 5 Ask "How man Example with 138 + 6	y 5s in 40?" 5+5+5+5+5+5+5+5 = 81 0 5 10 15 20 25 30 35 40 remainder.	n a remainder of 2



- 4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).
- 4 goes into 16 four times.

Step 1—a remainder in the ones

4 goes into 5 once, leaving a remainder of 1.

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- 8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).
- 8 goes into 32 four times  $(3,200 \div 8 = 400)$
- 8 goes into 0 zero times (tens).
- 8 goes into 7 zero times, and leaves a remainder of 7.

Step 1 continued...

When dividing the ones, 4 goes into 7 one time. Multiply  $1 \times 4 = 4$ , write that four under the 7, and subract. This finds us the remainder of 3.

Check:  $4 \times 61 + 3 = 247$ 

When dividing the ones, 4 goes into 9 two times. Multiply  $2 \times 4 = 8$ , write that eight under the 9, and subract. This finds us the remainder of 1.

Check: 4 × 402 + 1 = 1,609

### Step 2—a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o	t o	t o 2 9
2)58	2)58 -4 1	2)5 <mark>8</mark> -4↓ 1 <mark>8</mark>
Two goes into 5 two times, or 5 tens  • 2 = 2 whole tens but there is a remainder!	To find it, multiply 2 × 2 = 4, write that 4 under the five, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o	t o	t o
2 9 2 ) 5 8	2 9 5 8	2 <del>2 9</del> 2 ) 5 8
- <u>4</u>	<u>- 4</u>	<u>-4</u> 18
	<u>- 1 8</u>	<u>- 1 8</u>
Divide 2 into 18, Place 9 into the	Multiply 0 x 2 = 49, write that 49	The division is over since there are
quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract.	The division is over since there are no more digits in the dividend. The quotient is 29.

### **Long Division**

Step 2—a remainder in any of the place values

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
h t o 1 2 ) 2 7 8	2)278 -20 0	2)278 -2↓ 07
Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	Next, drop down the 7 of the tens next to the zero.
Divide.	Multiply & subtract.	Drop down the next digit.
1 3 2 ) 2 7 8 -2 0 7	13 2)278 -2 07 -6	13 2)278 -2 07 -6 18
Divide 2 into 7. Place 3 into the quotient.	Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
13 <mark>9</mark> 2)278 -2 07 -6	139 2)278 -2 07 -6 18 -18	139 2)278 -2 07 -6 18 -18
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	There are no more digits to drop down. The quotient is 139.

